Large-scale layout of facilities using a heuristic hybrid algorithm

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In this article the layout of facilities is formulated as a quadratic assignment problem and then investigated using a heuristic hybrid algorithm. The algorithm presented is a hybrid in the sense that it incorporates the short-term memory aspects of tabu search and the probabilistic aspects of simulated annealing. When applied to some large-scale problems the hybrid algorithm produces results superior to the equivalent tabu search algorithm in both solution quality and execution time. Furthermore, for a particular standard problem, the mean solution quality obtained using the hybrid algorithm compares favorably with the best individual value known to date. Hence, the hybrid algorithm is shown to be an effective computational technique for solving large-scale facilities layout problems.

Keywords: facilities layout, quadratic assignment, simulated annealing, tabu search

1. Introduction

The layout of facilities may be formulated as a quadratic assignment problem (QAP) and, as such, is an NP-complete combinatorial optimization problem,[1] which was originally formulated by Koopmans and Beckmann[2] to mathematically model the assignment of economic activities to geographical locations. A QAP involves, in the broadest sense, the determination of the optimal assignment of objects (here facilities) to locations and requires quantifiable measures of both distance and interaction between objects. In industrial contexts QAPs frequently result from management problems in which, typically, the aim is to minimize financial costs or maximize operational efficiency (for applications see Refs. 3–6).

Over the past few decades several techniques have been proposed to obtain optimal assignments (here layouts), for example, explicit and implicit enumeration and branch and bound methods. For reviews of these and other associated procedures, see Refs. 7–10. However, because NP-complete problems cannot currently be solved by any known polynomial-time algorithm, suboptimal procedures must be used with problems in which the number of objects and locations exceeds 15; these procedures include branch and bound related methods,[11,12] simulated annealing,[13–15] and tabu search.[16-18]

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The aims of this article are to present a heuristic algorithm for facilities layout that combines aspects of simulated annealing and tabu search, and to investigate the performance of the algorithm in terms of quality of results and execution time when applied to some large-scale problems.

The format of the rest of this article is as follows: Section 2 contains the mathematical formulation of the facilities layout problem in terms of a QAP. The hybrid algorithm is presented in Section 3. A description of the numerical experiments and a discussion of the results obtained are given in Section 4 and Section 5, respectively. Section 6 contains the conclusions of the investigations.

2. Formulation

The basic QAP can model quite sophisticated real-world situations in the manner now described in general terms. With n objects and n locations it is necessary to define a $(n \times n)$ matrix D with elements $d(i,j)$ that specify the distance between locations i and j. Furthermore, matrix C, of the same dimensions as D and with elements $c(k, l)$, gives the interaction, or connectivity, between objects k and l. Both objects and locations are labelled 1 to n, and P, a one-dimensional array containing the numbers 1 to n, is used to indicate the location of objects. For example, if the yth element of P is x then the position of object x is location y, denoted by $p(x) = y$. The set of all feasible solutions, i.e., permutations of P, constitutes a solution-space, S, which may have as many as $N!$ elements. However, here S will be somewhat smaller because C and D are taken to be symmetric with $c(i, i) = d(i, i) = 0$, $i = 1, 2 \ldots n$. In this situation, the cost function to be minimized, $Q(P)$, is given by

$$Q(P) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c(i, j)\, d(p(i), p(j)) \tag{1}$$

Equation (1) is an expression of the total interconnection distance between all objects, which, depending on what the elements of C actually represent, may, for example, translate to a financial quantity.

3. Hybrid algorithm

In this article, features of two recent heuristics, simulated annealing and tabu search, are combined to give a hybrid algorithm. Considered individually, simulated annealing and tabu search are both iterative improvement algorithms in that they both start from some feasible solution and attempt to determine a better assignment of objects to locations by means of a permutation of the current assignment. Tabu search, however, uses a semi-intelligent approach to search the solution space, in contrast to the probabilistic search technique of simulated annealing. Both algorithms are characterized by their ability to escape local optima.

Tabu search is a deterministic technique that has links with ideas in the field of artificial intelligence. For detailed source information see Refs. 19–21; here a general outline is given.

With tabu search the movement through solution space is via neighborhoods of the current solution (i.e., assignments of objects to locations). If P denotes the current solution then a suitable neighborhood is the set of solutions generated by the pairwise exchange of any two elements of P. Furthermore, if the pairwise exchange neighborhood of P is denoted by $N(P)$ (with $P \notin N(P)$) then the search moves to the allowed solution in $N(P)$ with the lowest cost, P', for example. Whether or not a solution is allowed is determined by reference to a one-dimensional array, called the tabu list, because it contains (temporarily) forbidden, or tabu, solutions. Once the search has moved to P' (i.e., P' is accepted) it is included in the tabu list, which acts as a short-term memory in that it contains a specified number of the most recently accepted solutions. The number of solutions in the tabu list is called the tabu list length and is maintained on the basis of a first-in first-out queue. Throughout the search the current (overall) minimum cost value, i.e., current best cost, is recorded.

The procedure of generating a neighborhood, accepting the allowed (i.e., nontabu) solution with the lowest cost and then updating the tabu list is the most basic tabu search technique and can be repeated until, for example, a maximum number of moves is reached or a specified computation time is exceeded. If, during this procedure a local optimum (here minimum) is encountered then the search may be forced out of the locality by means of reference to the tabu list. An additional merit of the tabu list is the prevention, to a degree, of cycling within a small subset of the solution space.

However, as the problem size, n, increases the time taken to search entire neighborhoods may become prohibitively large; a useful modification to the basic tabu search technique comprises the acceptance, when it occurs, of the first nontabu neighborhood solution encountered in the neighborhood of P for which $Q(P_N) < Q(P)$, where $P_N \in N(P)$. This modification, which requires $Q(P_N)$ to be evaluated for all $P_N \in N(P)$ as a last resort only, has been found to save time without undue loss of solution quality and will be adopted in this article.[20,21]

Retaining the neighborhood and short-term memory aspects of tabu search, a hybrid algorithm can be created by utilizing the probabilistic aspects of simulated annealing.

Simulated annealing was originally proposed by Kirkpatrick et al.[22] and independently by Cerny,[23] and was based on a Monte Carlo method proposed by Metropolis et al.[24] to simulate the changes in atomic configuration as a heated substance cools to thermal equilibrium. Further information, including the theoretical basis for simulated annealing, can be found in Refs. 25–27. In this article tabu search is hybridized by means of the following acceptance conditions for nontabu solutions; accept if

$$\text{either} \quad Q(P_N) \le Q(P) \tag{2}$$

$$\text{or} \quad r < \exp\left[\frac{Q(P) - Q(P_N)}{T}\right] \tag{3}$$

Inequality (2) states that a nondetrimental solution is accepted with probability unity, whereas inequality (3) indicates that a detrimental solution (i.e., one with $Q(P_N) > Q(P)$) is accepted with probability $\exp[(Q(P) - Q(P_N))/T]$. In inequality (3), r is a random number in the range (0, 1) and T is a control parameter called, by analogy, the temperature. Initially the temperature is set sufficiently large and then allowed to decrease in accordance with a specified cooling schedule.

In this article the analogy between the combinatorial optimization problem and a physical system undergoing annealing is continued to enable a plausible cooling schedule based on Newton's law of cooling to be obtained.

$$T_k = T_{k-1}\left[\frac{T_f}{T_i}\right]^{1/m} \qquad k = 1, 2, \ldots, m \tag{4}$$

where, in the physical system, T is recorded at m equal time intervals of size h; T_i and T_f denote the initial and final temperatures, respectively; and $T_k = T(kh)$ with $T_0 = T_i$.

With a set of random feasible solutions, S', suitable values of T_i and T_f can be obtained by solving $r = \exp[\text{mean}\,[Q(P_i) - Q(P_j)]/T]$ for T with $r = 0.9$ and 0.001, respectively, where $Q(P_i) < Q(P_j)$ and $P_i, P_j \in S'$. Hence, knowing the ratio of the initial and final temperatures and the number of temperature recordings, the temperature at each stage of cooling, T_k, $k = 1, 2 \ldots m$, may be obtained from equation (4), the "natural" cooling schedule. In the combinatoric problem, the time interval h corresponds to a fixed number of trials, i.e., pairwise exchanges (taken to be 1000), and m is the number of discrete temperatures (taken to be 50).

4. Numerical experiments

In order to investigate the applicability of the hybrid algorithm to the layout of facilities, various experiments were performed on four large-scale problems (in the sense that $n > 15$). The smallest problem had size $n = 30$ and was the largest of a group originally investigated by Nugent et al.[8] The other three problems had sizes $n = 50$, 75, and 100, and were generated randomly.

In Nugent's problem ($n = 30$) the locations were represented as unit squares in a (5×6) configuration, and the distances between facilities were given by the rectilinear measure. Details of the facility connectivity values, location distances, and starting solutions are not presented here but can be found in Ref. 8.

The three randomly generated problems also had unit square locations, in configurations of (5×10), (5×15), and (10×10), and, as with Nugent's problem, rectilinear distances were used. Furthermore, the facility connectivity values were generated using the same range of values and density of nonzero values as Nugent's problem. Also, to remain consistent with Nugent's approach, five starting solutions were (randomly) generated for each problem.

Investigations with (pure) tabu search have shown that both solution quality and execution time increase as the tabu list length, L, increases, and that a suitable trade-off between solution quality and execution time occurs when L is proportional to n.[20,21] For all problems in this study $L = n$.

All numerical experiments were conducted on a VAX–11/785 computer using VAX-FORTRAN. In all problems the search was allowed to continue until there was no change in the current best cost for 50 consecutive neighborhoods. Based on five starting solutions, the mean best costs and associated mean execution times, obtained when the hybrid algorithm was applied to each of the four problems, are given in *Table 1*.

For comparison purposes, the probabilistic acceptance condition of the hybrid algorithm was suppressed to yield a pure tabu search algorithm; all numerical experiments were then repeated with results shown in *Table 2*.

5. Discussion

Inspection of *Table 1* and *Table 2* reveals, for all problem sizes considered, the superiority of the hybrid algorithm over the tabu search algorithm in terms of mean best cost and mean execution time. Furthermore, on closer observation it is seen that the hybrid algorithm starts with a small savings in mean execution time (for $n = 30$), but then gives an ever-increasing savings as n increases. In contrast the improvement in mean best cost, although always positive, varies erratically as n increases and is rather small for $n = 75$.

Table 1 Mean costs and execution times (in seconds) for the hybrid algorithm

Problem size n	Mean best cost	Mean execution time
30	3088	2.9
50	39308	81.4
75	56608	101.1
100	63925	231.3

Table 2 Mean costs and execution times (in seconds) for tabu search

Problem size n	Mean best cost	Mean execution time
30	3101	3.8
50	39996	183.6
75	56615	249.8
100	64163	410.4

The facilities layout problem (i.e., QAP) has recently been studied, using tabu search and simulated annealing individually, by several authors.[14–18] Each author presents computational results and the merits of the particular implementation used. Unfortunately, however, it is not easy to compare results between authors and techniques on a like-for-like basis; for example, unlike tabu search, simulated annealing does not require neighborhoods. Also, for example, the tabu search implementation of Skorin–Kapov[17] (called tabu navigation) enables the search to return, after a specified number of neighborhoods, to the current best solution and then continue from this base in a new "direction."

Notwithstanding the problem of direct comparison of results, the best costs found by various authors for Nugent's problem ($n = 30$), as reported by Taillard,[18] are given in *Table 3*. As shown in *Table 3*, the mean best cost obtained with the hybrid algorithm (cf. *Table 1*) compares favorably with previous best found values. In fact, the hybrid algorithm gave results that "on average" were only 26 (0.85%) above 3062, which is considered to be the global minimum.[18]

Attention is given to Nugent's problem ($n = 30$) because it is the largest in a series with well-known distance and connectivity values, and, importantly, it has five starting solutions documented. For comparison purposes it is unfortunate that this situation does not exist with larger problems.

6. Conclusion

In this article an algorithm has been presented for facilities layout (QAP formulation) that incorporates the short-term memory aspects of tabu search and the

Table 3. Best costs obtained by various investigators for Nugent's problem ($n = 30$)

Investigator(s)	Best cost
Nugent et al.[8]	3189
ᶜIshafei[5]	3125
Burkard and Rendl[13]	3074
Connolly[27]	3062
Skorin–Kapou[17]	3062
Taillard[18]	3062

probabilistic aspects of simulated annealing. Numerical experiments with the hybrid algorithms on four large-scale problems yielded results which, when compared to the equivalent tabu search algorithm (i.e., with suppressed probabilistic aspects), are superior in both solution quality and execution time. Furthermore, although direct comparison with results obtained by other investigators is not straightforward, the mean best cost obtained with the hybrid algorithm when applied to Nugent's problem ($n = 30$) was found to be competitive with the best individual values found in recent studies that used simulated annealing and tabu search separately. Based on the results obtained it is concluded that the present hybrid algorithm is an effective computational technique for solving large-scale facilities layout problems.

Finally, it is noted that, on reflection, the superiority of the hybrid algorithm over its equivalent tabu search algorithm may not be unexpected; the utilization of the probabilistic acceptance condition allows the search to move relatively more quickly (particularly in the early stages) from neighborhood to neighborhood. In other words, this increased mobility allows the search to move quickly through solution space and hence enables the hybrid algorithm to have the capability to find good-quality solutions faster. Typical convergence curves are shown in *Figure 1* in which the broken and continuous lines give the history of current best costs obtained by the hybrid and (equivalent) tabu search algorithms, respectively, and clearly indicate the superiority of the former throughout the minimization process. For information, the curves in *Figure 1* are based on results obtained from two of the numerical experiments with $n = 100$. The initial costs were 78096 and the final costs and execution times were 63987 in 214.5 s for the hybrid algorithm (denoted by a circle) and 64213 in 406.3 s, for tabu search (denoted by a square).

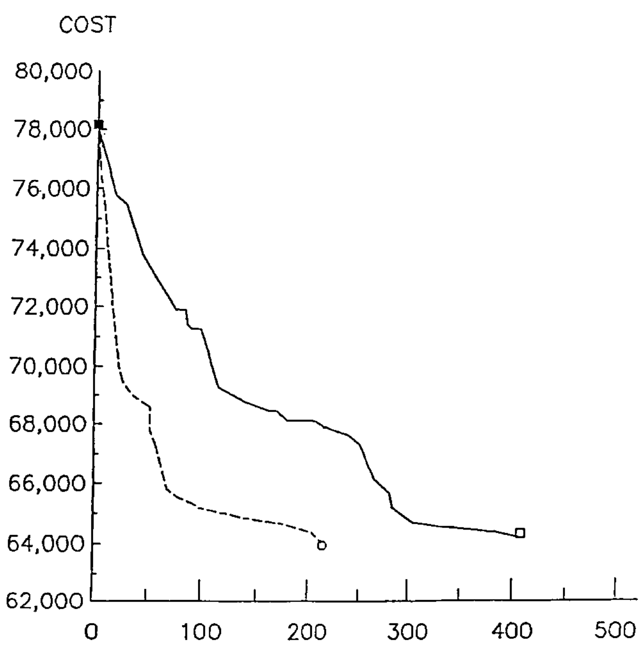

Figure 1. Typical convergence curves for the hybrid algorithm (broken line) and tabu search (continuous line).

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